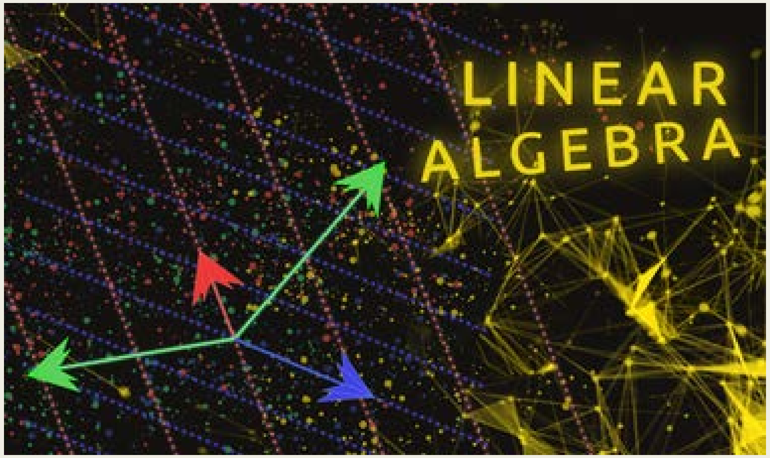
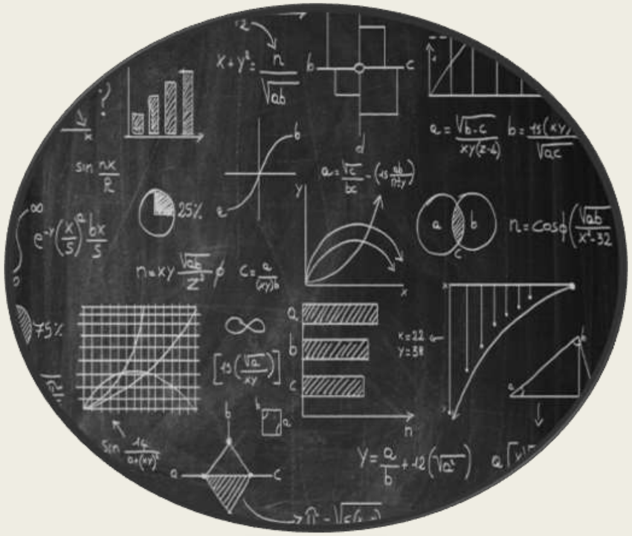


LINEAR ALGEBRA
LA in CS

Nooshin Maghsoodi
Noshirvani University



➢ *Introduction*

When you

- take a digital photo with your phone or transform the image in Photoshop,
- play a video game,
- watch a movie with digital effects,
- do a web search or
- make a phone call,

you are using technologies that build upon linear algebra.

➢ *Why Linear Algebra is important ?*

Linear algebra is built on two basic elements:

- the matrix and
- the vector.

It converts large number of problems to matrix and thus we solve the matrix.

➢ *Why this course?*

- **Goal**: That vectors and matrices and orthogonal projection and eigenvalues and .... become things you interact with and use almost as easily as you use a pencil or a computer or a while-loop or recursion.
- **Aim**: To understand some very cool but more advanced concepts/applications:
  - *k-Means Clustering, Principal Components Analysis, image deblurring, image*
  - *compression, ...*

دانشگاه صنعتی نوشیروانی بابل

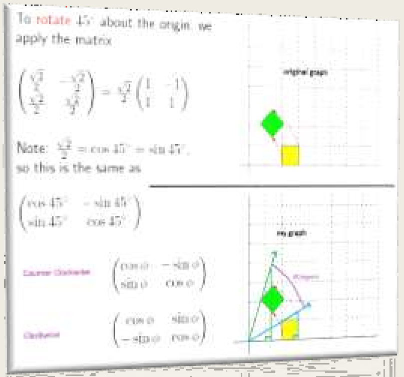
➢ *Uses of Linear Algebra in CSE*

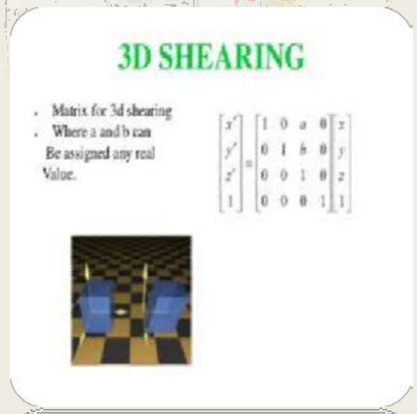
❑ ***Linear Algebra in computer science can broadly divided into two categories:***

1) Linear Algebra for spatial quantities.

Here you're dealing with 2-, 3-, or 4-dimensional vectors and you're concerned with rotations, projections, and some other matrix operations that have spatial interpretation.

This is the kind of linear algebra that comes up, for example, in computer graphics and physics simulations.

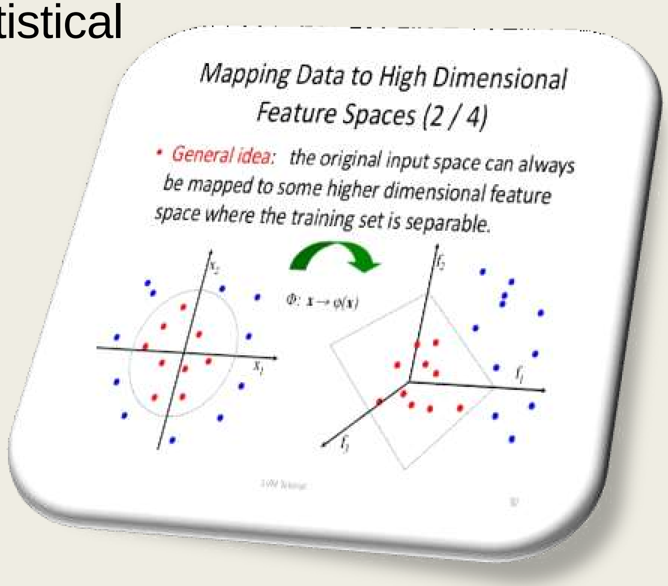




2) Linear Algebra for statistics.

Here you're dealing with vectors in high-dimensional spaces that have no particular spatial interpretation and you're interested in matrix decompositions and so on.

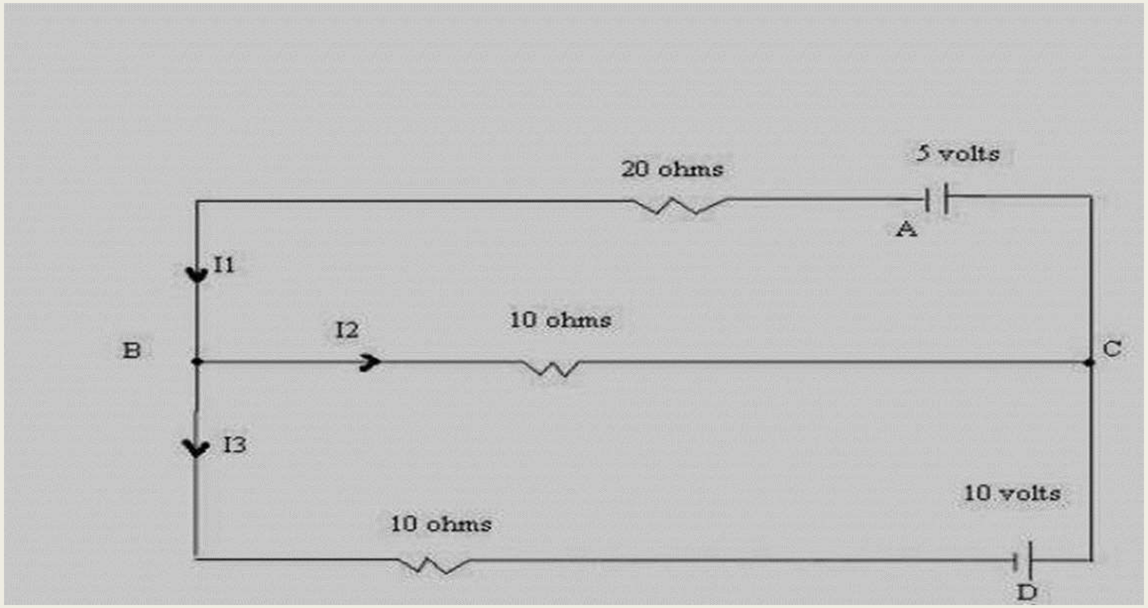
This domain includes signal processing, statistical machine learning, and compression.



دانشگاه صنعتی نوشیروانی بابل

➢ *Linear Algebra in Network Models*

Determine the currents I1, I2, and I3 for the following electrical network:



By Kirchhoff's Law:

$$\begin{cases} I_1 - I_2 - I_3 = 0 \\ -10I_2 + 10I_3 = 10 \\ 20I_1 + 10I_2 = 5 \end{cases} \Rightarrow \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -10 & 10 & 10 \\ 20 & -10 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0.4 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 1 & 0.7 \end{bmatrix} \Rightarrow I_1 = 0.4, \quad I_2 = -0.3, \quad I_3 = 0.7$$

➢ *Linear Algebra in Cryptography*

Encryption and decryption require the use of some secret information, usually referred to as a key.

Example Let the message be

"PREPARE TO NEGOTIATE"

We assign a number for each letter of the alphabet. Thus the message becomes:

P	R	E	P	A	R	E	*	T	O	*	N	E	G	O	T	I	A	T	E
16	18	5	16	1	18	5	27	20	15	27	14	5	7	15	20	9	1	20	5

Since we are using a 3 by 3 matrix, we break the enumerated message above into a sequence of 3 by 1 vectors:

$$\begin{bmatrix}16\\18\\5\end{bmatrix} \quad \begin{bmatrix}16\\1\\18\end{bmatrix} \quad \begin{bmatrix}5\\27\\20\end{bmatrix} \quad \begin{bmatrix}15\\27\\14\end{bmatrix} \quad \begin{bmatrix}5\\7\\15\end{bmatrix} \quad \begin{bmatrix}20\\9\\1\end{bmatrix} \quad \begin{bmatrix}20\\5\\27\end{bmatrix}$$

By multiplying encoding matrix to this matrix we will encrypt the msg.

$$\begin{bmatrix}-3 & -3 & -4\\0 & 1 & 1\\4 & 3 & 4\end{bmatrix} \begin{bmatrix}16 & 16 & 5 & 15 & 5 & 20 & 20\\18 & 1 & 27 & 27 & 7 & 9 & 5\\5 & 18 & 20 & 14 & 15 & 1 & 27\end{bmatrix} \begin{bmatrix}-122 & -123 & -176 & -182 & -96 & -91 & -183\\23 & 19 & 47 & 41 & 22 & 10 & 32\\138 & 139 & 181 & 197 & 101 & 111 & 203\end{bmatrix}$$

Now to decrypt the msg we have to multiply this matrix to Inverse of encoding matrix

- The inverse of this encoding matrix, the decoding matrix, is:

$$\begin{bmatrix} 1 & 0 & 1 \\ 4 & 4 & 3 \\ -4 & -3 & -3 \end{bmatrix}$$

- Multiplying again by this matrix we will get our Msg.

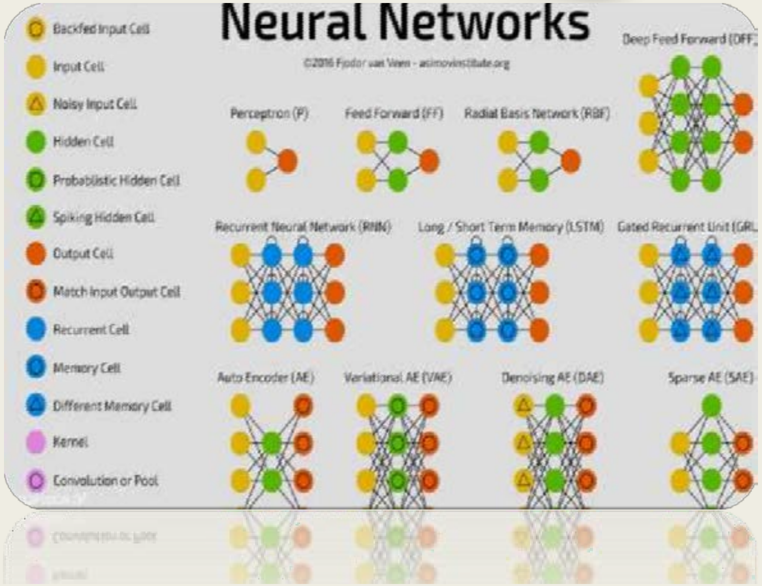
Very important in Machine Learning. For example :

- ❑ Dimensionality reduction;

 (e.g. Principal component analysis)

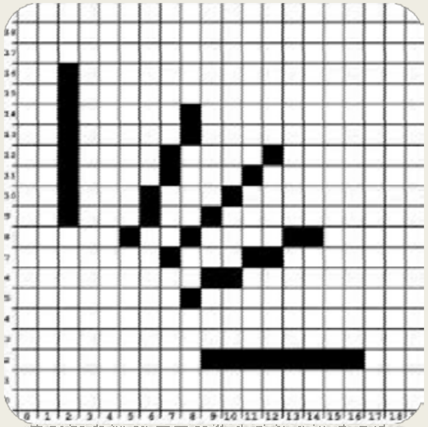
- ❑ Clustering;
- ❑ Classification;
- ❑ Prediction;
- ❑ Recommender systems

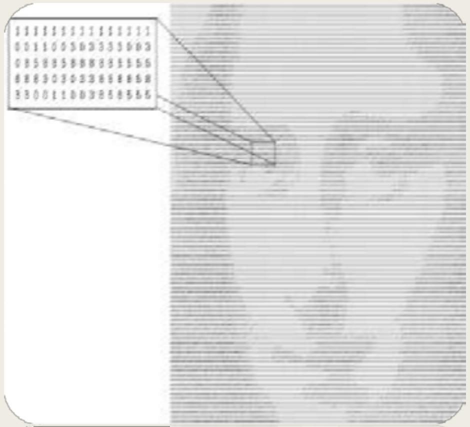
 (e.g Collaborative filtering) etc..



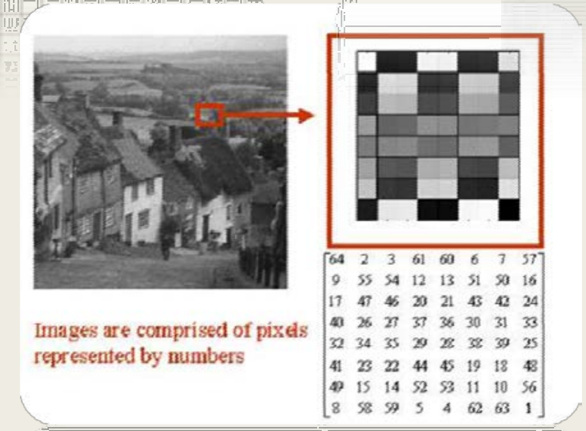
➢ *Linear Algebra Computer Graphics*

✓In computer graphics every element is represented by a MATRIX.

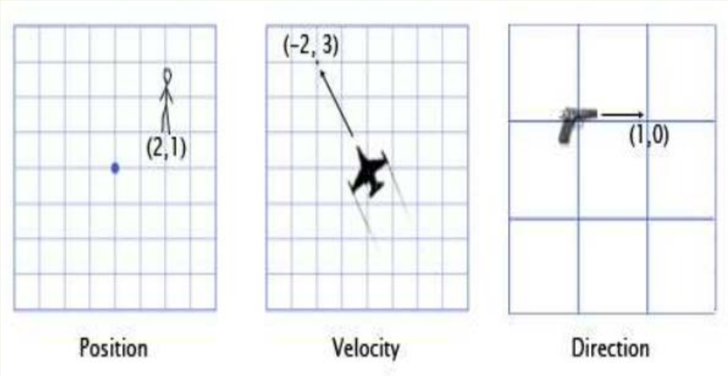




✓ All images can be represented in matrix format.



➢ *Linear Algebra in video games*



Linear algebra is the study of vectors.

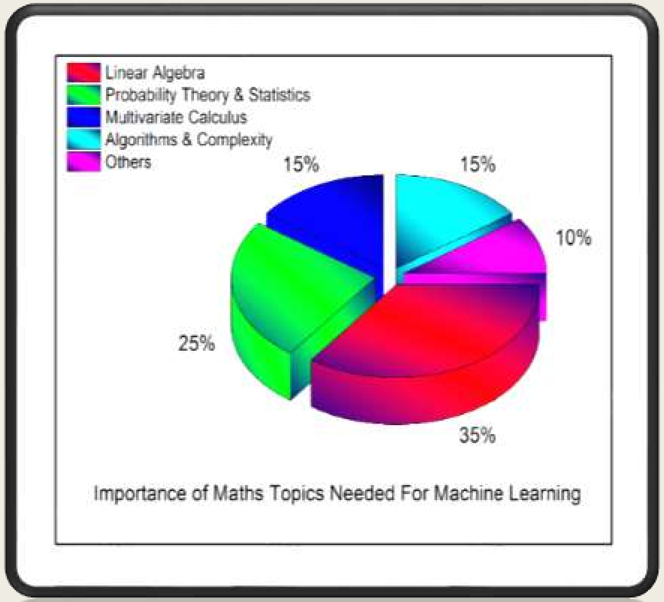
If your game involves the position of an on-screen button, the direction of a camera, or the velocity of a race car, you will have to use vectors.

The better you understand linear algebra, the more control you will have over the behavior of these vectors.

In games, vectors are used to store positions, directions, and velocities. Here are some 2-Dimensional examples:

- The position vector indicates that the man is standing two meters east of the origin, and one meter north.
- The velocity vector shows that in one minute, the plane moves three kilometers up, and two to the left.
- The direction vector tells us that the pistol is pointing to the right.

➢ Conclusion



- ❑ There are so many application of Linear Algebra in Computer Science.
  - ❑ From simple circuit solving to large web engine algorithms.

- ❑ The heart beat of computer science is in linear algebra.