Sec 4.6 Rank

Nooshin Maghsoodi

Noshirvani University

These slides are adapted from Linear Algebra course in UESTC

Outline

Outline

Row Space

Definition

If A is an $m \times n$ matrix. The set of all linear combinations of the row vectors is called the row space of A and is denoted by Row$A$.

Row Space

Definition

If A is an $m \times n$ matrix. The set of all linear combinations of the row vectors is called the row space of A and is denoted by Row$A$.

- Row$A$ is a subspace of $\mathbb{R}^n$.
- One can write Row$A$ as ColA^T

Row Space

Theorem

- If two matrices A and B are row equivalent, then their row spaces are the same.

Row Space

Theorem

- If two matrices A and B are row equivalent, then their row spaces are the same.
- If B is in echelon form, the nonzero row of B form a basis for the row space of A as well as for that of B.

Example

Find the dimensions of the row space, null space and the column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

Example

Find the dimensions of the row space, null space and the column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Example

Find the dimensions of the row space, null space and the column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

dimRow$A$=2, dimNul$A$=3, and dimCol$A$=2.

Outline

Rank and the Rank Theorem

Definition

The **rank** of A is the dimension of the column space of A.

Rank and the Rank Theorem

Definition

The **rank** of A is the dimension of the column space of A.

Theorem

- If A is an $m \times n$ matrix, then

$$\text{rank} A + \dim \text{Nul} A = n$$

Rank

Example

- If A is a 6×7 matrix with a two-dimensional null space, what is the rank of A?

Rank

Example

- If A is a 6×7 matrix with a two-dimensional null space, what is the rank of A?
- Could a 3×6 matrix have a two-dimensional null space?

Outline

The Invertible Matrix Theorem (Continued.)

- m. The columns of A form a basis of $\mathbb{R}^n$.
- n. $\text{Col}A = \mathbb{R}^n$
- o. $\text{dimCol}A = n$
- p. $\text{rank}A = n$
- q. $\text{Nul}A = \{0\}$
- r. $\text{dimNul}A = 0$