Characterizations of Invertible Matrices

Nooshin Maghsoodi

Noshirvani University

These slides are adapted from Linear Algebra course in UESTC

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ



Invertible Matrix Theorem

(ロ)、(型)、(E)、(E)、 E) の(()

The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are **equivalent**. That is, for a given A, the statements are either all true or all false.

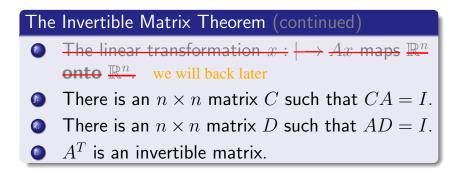
- A is an invertible matrix.
- A is row equivalent to the $n \times n$ identity matrix.
- A has n pivot positions.
- The equation Ax = 0 has only the trivial solution.

The Invertible Matrix Theorem (continued)

- The columns of A form a linearly independent set.
- we will back later to item f
- Solution Ax = b has at least one solution for each $b \in \mathbb{R}^n$.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

• The columns of A span \mathbb{R}^n .



▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のへで

An $n \times n$ matrix A is said to be **invertible** if there is an $n \times n$ matrix C such that

$$CA = I$$
 and $AC = I$

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ □臣 ○のへ⊙

An $n \times n$ matrix A is said to be **invertible** if there is an $n \times n$ matrix C such that

$$CA = I$$
 and $AC = I$

Let A and B be square matrices. If AB = I, then A and B are both invertible, with B = A⁻¹ and A = B⁻¹.
Definition: Rank of a matrix

The number of pivot columns in the matrix