Characterizations of Invertible Matrices

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Outline

1. Invertible Matrix Theorem

The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are **equivalent**. That is, for a given A, the statements are either all true or all false.

a. A is an **invertible** matrix.
b. A is **row equivalent** to the $n \times n$ identity matrix.
c. A has n pivot positions.
d. The equation $Ax = 0$ has **only the trivial** solution.

The Invertible Matrix Theorem (continued)

- e. The columns of A form a linearly independent set.
- f. we will back later to item f
- g. The equation $Ax = b$ has **at least** one solution for each $b \in \mathbb{R}^n$.
- h. The columns of A span $\mathbb{R}^n$.

The Invertible Matrix Theorem (continued)

- i. ~~The linear transformation $x : \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.~~ we will back later
- j. There is an $n \times n$ matrix C such that $CA = I$.
- k. There is an $n \times n$ matrix D such that $AD = I$.
- l. A^T is an invertible matrix.

Invertible Matrices

An $n \times n$ matrix A is said to be **invertible** if there is an $n \times n$ matrix C such that

$$CA = I \quad \text{and} \quad AC = I$$

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- Let A and B be **square** matrices. If $AB = I$, then A and B are both invertible, with $B = A^{-1}$ and $A = B^{-1}$.

Definition: Rank of a matrix
The number of pivot columns in the matrix